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COMMENT

The inertial mass and anomalous internal angular momentum of a phase-locked cavity

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Abstract. It has been shown that phase-locked cavities have a quantised transfer function (Jennison 1979) and that they account for the property of inertia (Jennison and Drinkwater 1977). The analysis in this paper shows that only half of the rest energy of a phase-locked cavity contributes actively to the inertial force. The moment of inertia of such a system about its own axis is only half that of a classical rigid body of the same total rest mass distributed in the same region of space. If a phase-locked cavity is used as a model of a fundamental particle then it is concluded that the particle is a fermion with a rest mass corresponding to its total rest energy.

In their derivation of Newton's second law, Jennison and Drinkwater (1977) utilised the relativistic transformation of a light complex (Einstein 1905), by which means they were able to generalise the problem without entering into specific details of the mechanism for containing the radiation. Einstein's derivation of the total energy of a light complex assumes that it is unbound except by its own wavefront and although the introduction of stationary, perfectly reflecting boundaries cannot affect the energy of the light itself it does imply that if one analyses the total system of the light plus imposed boundaries, one must also include in the analysis the energy required to maintain these impositions. If a particle is formed entirely from an electromagnetic wave system then one must exercise considerable care in applying those mechanical concepts which imply in their formulation the presence of restoring forces which, in turn, imply the existence of a potential source of energy, this is revealed in the present analysis which uses the concept of radiation pressure within the cavity.

We will now consider the specific case of the $\lambda/2$ standing wave system shown in figure 1(b) of Jennison (1978); this standing wave has zero electric field at the centre. We shall consider that the system has unit cross-sectional area and that boundaries of negligible mass hold the system together, the source of the binding energy may be provided by a spinning configuration of the electromagnetic wave packet, as postulated by Jennison (1978) but a similar analysis also holds for phase-locked cavities of the types that have been constructed and for the elementary macroscopic case of a full wave system, with zero electric field at the centre, where the boundaries are conducting plates of negligible mass carrying equal and opposite charges. In this latter case λ should be substituted for $\lambda/2$ in equation (1) but the factor of two cancels in equation (4), since $\delta t = \lambda/c$, and thus equations (4) and (5) are unchanged.

Using units similar to Einstein (1905) the energy density of the travelling waves in the cavity at rest is $A^2/8\pi$ where A is the amplitude of either the electric or magnetic force. If the central node is caused to move, the energy density and the volume occupied by the wave system are both relativistically transformed. The cross-sectional area does not change but, as we are considering a phase-locked system, the length of the system to each side of the node is the effective length of the total travelling wave packet on each side.

We now consider that the central node is moved to the right at velocity v. Both of the component travelling waves to the right of the node have more energy and both of those to the left have less energy than when at rest since the boundaries at each end redirect the radiation within the time taken to complete the feedback loop. Thus the total energy of the system to an observer on the moving node is given by the transformed potential energy plus the transformed energy density times the transformed total wavelength to the right plus the transformed energy density times the total transformed wavelength to the left.

$$\begin{aligned} E'_{\mathrm{T}} &= E'_{\mathrm{P}} + E'_{\mathrm{WR}} + E'_{\mathrm{WL}} \\ &= E'_{\mathrm{P}} + \frac{A^2}{8\pi}\left(\frac{1+v/c}{1-v/c}\right) \cdot \frac{\lambda}{2}\left(\frac{1-v/c}{1+v/c}\right)^{1/2} + \frac{A^2}{8\pi}\left(\frac{1-v/c}{1+v/c}\right) \cdot \frac{\lambda}{2}\left(\frac{1+v/c}{1-v/c}\right)^{1/2} \\ &= E'_{\mathrm{P}} + \frac{A^2\lambda}{16\pi}\left[\left(\frac{1+v/c}{1-v/c}\right)^{1/2} + \left(\frac{1-v/c}{1+v/c}\right)^{1/2}\right] \end{aligned} \tag{1}$$

where E'_{P} is the transformed potential energy required to hold the system together, E'_{WR} is the wave energy to the right of the node and E'_{WL} is that to the left.

The radiation pressure (Einstein 1905) at the moving node from the wave system on the left is

$$P'_{\mathrm{L}} = \frac{2A^2}{8\pi}\left(\frac{1-v/c}{1+v/c}\right)$$

and that from the wave system on the right is

$$P'_{\mathrm{R}} = \frac{2A^2}{8\pi}\left(\frac{1+v/c}{1-v/c}\right)$$

The difference in these two expressions gives the force $\delta F'$ on the unit area at the node

$$\delta F' = \frac{A^2}{4\pi}\left(\frac{1+v/c}{1-v/c} - \frac{1-v/c}{1+v/c}\right). \tag{2}$$

From (1),

$$\frac{A^2}{4\pi} = 4(E'_{\mathrm{T}} - E'_{\mathrm{P}}) \Big/ \lambda\left[\frac{1+v/c^{1/2}}{1-v/c} + \frac{1-v/c^{1/2}}{1+v/c}\right]$$

Therefore

$$\begin{aligned} \delta F' &= \frac{4}{\lambda}(E'_{\mathrm{T}} - E'_{\mathrm{P}})\left[\left(\frac{1+v/c}{1-v/c}\right)^{1/2} - \left(\frac{1-v/c}{1-v/c}\right)^{1/2}\right] \\ &= \frac{8(E'_{\mathrm{T}} - E'_{\mathrm{P}})}{\lambda(1-v^2/c^2)^{1/2}}\frac{v}{c}. \end{aligned} \tag{3}$$

This analysis is based entirely on elementary relativistic physics with no additional postulates, in particular we have not postulated that the wave system to the right of the node, loops around to appear as the wave system on the left in accordance with the model of an electron suggested in the conclusions to the paper by Jennison (1978). If this postulate is made, then there are not separate journey times for the echoes from the disturbed wave systems to the right and left of the node and a unique time exists for the disturbances travelling each way around the loop. This in turn provides a rigorous solution to the following calculation where, in the absence of such a postulate, we will revert to first order by taking a mean journey time of the two disturbances, $\delta t = \lambda/2c$, whence, to first order:

$$\begin{aligned}\delta F &= 2(E_{\mathrm{T}} - E_{\mathrm{P}})(2v/\delta t)/c^2 \\ &= 2(E_{\mathrm{T}} - E_{\mathrm{P}})a/c^2 \end{aligned} \tag{4}$$

where a is the acceleration established over a complete feedback cycle. The first-order treatment is justified since inertial effects are large at relative velocities as low as 1 ms^{-1} when v^2/c^2 is only 10^{-17}. Now in the rest state, the wave energy equals the binding energy and they together comprise the total energy, hence

$$F = E_{\mathrm{T}}a/c^2 = m_0 a. \tag{5}$$

Thus we derive Newton's Law, but this analysis is enlightening in that equation (4) shows that only half of the total energy comprising the inertial mass contributes actively to the inertial force, the law of inertia would be twice as efficient if the potential energy also contributed to the inertial force of a phase-locked cavity, i.e. if the transformation of E_{P} had a first-order component. Alternatively, it will be seen that if E'_{P} is not included in equation (1), we obtain $\delta F = 2ma$, which contradicts observation.

Formally the result is entirely in agreement with d'Alembert's Principle (1742) in that the binding forces make no contribution to the external dynamics of the system, but this alone might lead one to expect a diminished inertial mass as half the rest-energy is associated with the binding force. It is therefore remarkable that the factor of two multiplying the Doppler-like contribution of the active component precisely compensates for this in the resulting expression for the inertial mass of the system. The only alternative attempt to explain inertia, that which is based on the postulate of Mach's Principle, does not differentiate between the two internal components, for it is an entirely qualitative argument which in no way accounts for the detailed behaviour of accelerated matter (cf Jennison and Drinkwater 1977).

The inertial rest mass incorporated in Newton's second law is only applicable when a complete particle has been formed as a phase-locked system. This complete system can interact with external forces entirely in accordance with the laws of mechanics and the total mass $m_0 = E_{\mathrm{T}}/c^2$ is available to produce the observed reaction to an impressed force. In contrast, if we endeavour to apply the classical inertial laws to the constituent field systems of a closed loop wave packet (a phase-locked cavity), we do not have a situation where there are complete subsystems of ready made particles to which $\delta m_0 = \delta E/c^2$ may be applied but only a flux of energy without rest mass and we may only employ half of this total energy in establishing the active component. The other half of the field energy is required, within the fundamental structure of the mechanics responsible for the law, to assume the role of the passive binding energy in whatever form is required by the configuration of the complete system. Thus, for entirely classical reasons, the laws of mechanics, *in terms of total rest mass*, cannot be applied to the field systems which form the ingredients of elementary phase-locked systems, though they are perfectly valid for the behaviour of complete systems. This, of course, is entirely consistent with special relativity, for the electromagnetic energy travelling at a speed of c cannot be ascribed a rest mass without implying a relativistic mass of infinity, and infinite energy, in any inertial frame.

The concept of moment of inertia is based upon the concept of the complete inertial mass as it appears in Newton's law. If the concept is applied to the substance of a rotating phase-locked cavity, as distinct from an assemblage of such cavities, then only half of the total energy is actively operational, thus:

The moment of inertia of a phase-locked cavity about its own axis is half that of a classical rigid body of the same total rest mass distributed in the same region of space.

This is again consistent with the extreme case of d'Alembert's Principle and therefore with relativistic and classical mechanics excluding Boscovich's hypothesis.

Equation (1) applies generally to a whole range of models where radiation is trapped in small regions of space. Indeed, from the earlier work on relativistic rigidity and the proper unit of length, it does not seem possible to avoid the conclusion that the general principles must apply to all particles which conserve their proper mass within finite spatial regions, i.e. all particles having inertial rest mass and intrinsic life times $\gg h/m_0c^2$ (corresponding to very high cavity Q factors) whatever may be their detailed construction, provided only that they obey the principles of relativity.

If we now use a phase-locked cavity as a model of a fundamental particle and we wish to establish its internal angular momentum, we must reserve half of the total internal energy for the passive binding component so that the internal angular momentum is therefore only half that which would be given by considering the total internal energy of the system. This precise halving of the spin angular momentum shows that a phase-locked cavity behaves as a fermion with a rest mass corresponding to its total rest energy and it accounts very simply for the factor of two in the spin magnetic moment. Noting the contents of earlier papers (Jennison and Drinkwater 1977, Jennison 1978), it is remarkable that so many properties of phase-locked cavities are shared by so many fundamental particles.

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